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85. Proposed by E. W. MORRELL, A. M., Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

In turning a one-horse chaise within a ring of a certain diameter, it was observed that the outer wheel made two turns, while the inner wheel made but one. The wheels were each 4 feet high; and supposing them fixed at the distance of 5 feet on the axletree, what was the circumference of the track described by the outer wheel? From *Greenleaf's National Arithmetic.*

Solution by EDWIN R. ROBBINS, Master of Mathematics, Lawrenceville School, Lawrenceville, N. J.

Let r=radius, then $2\pi r$=circumference of inner track.

$r+5$=radius, then $2\pi(r+5)$=circumference of outer track.

From problem, $2(2\pi r)=2\pi(r+5)$, whence $r=5$.

Hence, outer track$=20\pi=62.83184+$ feet.

Also solved by *J. SCHEFFER, G. B. M. ZERR, F. R. HONEY, CHARLES C. CROSS, LEE WILCOX*, and *P. S. BERG.*

86. Proposed by EDGAR H. JOHNSON, Professor of Mathematics, Emory College, Oxford, Ga.

$\frac{1}{7}=.\dot{1}4285\dot{7}$; $\frac{1}{11}=.\dot{0}\dot{9}$; $\frac{1}{13}=.\dot{0}7692\dot{3}$; $\frac{1}{17}=.\dot{0}58823594117647\dot{}$.

Observe that if the numbers forming the first half of the repetend be added respectively to the numbers forming the second half of the repetend, the sum is in every case 9. What is the general law of which these are special cases?

I. Solution by O. W. ANTHONY, M. Sc. Instructor in Mathematics in Boys' High School, New York City.

Put $\dfrac{1}{x}=\dfrac{R}{10^u}\left[1+\dfrac{1}{10^u}+\dfrac{1}{10^{2u}}+\dfrac{1}{10^{2u}}+\text{ etc.}\right]$, where R is the sequence of digits in the repetend and u the number of digits.

Call R_1 the first half of the repetend expressed in whole numbers, and R_2 the second half.

Then $R_1+R_2=999\ldots\ldots=10^{\frac{1}{2}n}-1$.

Also $R=10^{\frac{1}{2}n}R_1+R_2$, $=10^{\frac{1}{2}n}R_1-R_1+10^{\frac{1}{2}n}-1$, $=(10^{\frac{1}{2}n}-1)(R_1+1)$.

$$\therefore\ \frac{1}{x}=\frac{(10^{\frac{1}{2}n}-1)(R_1+1)}{10^n}\cdot\frac{10^n}{10^n-1},\ \text{or}\ x=\frac{10^{\frac{1}{2}n}+1}{R_1+1}.$$

This gives the law of formation when the first part of the repetend is R_1.

Then, if $R_1=142$, $x=\dfrac{10^3+1}{143}=7$.

To find all repetends of six figures obeying the law, we proceed as follows:

$x=\dfrac{10^3+1}{R_1+1}=\dfrac{1001}{R_1+1}=\dfrac{13\times11\times7}{R_1+1}$. We must select such values for R_1 as will give x integral values.

If $R_1=12$, $x=77$.
If $R_1=10$, $x=9$.
If $R_1=6$, $x=143$.
If $R_1=76$, $x=13$.
If $R_1=142$, $x=7$.

If $R_1=90$, $x=11$.
Then $\frac{1}{7}=.142857\ldots\ldots$
$\frac{1}{11}=.090909\ldots\ldots$
$\frac{1}{13}=.076923\ \ldots\ldots$
$\frac{1}{77}=.0\dot{1}2987\ldots\ldots$
$\frac{1}{91}=.010989\ldots\ldots$
$\frac{1}{143}=.006993\ldots\ldots$

To find eight figured repetends obeying the law, we have,

$$x=\frac{10001}{R_1+1}.$$

To find ten figured repetends we have,

$$x=\frac{100001}{R_1+1},$$

and so on. In the last instance we may take as an illustration,

$R_1=10$, then $x=9091$, and $\frac{1}{9091}=.0001099989\ldots\ldots$

The problem reduces to resolving numbers into prime factors.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

There are some very peculiar properties connected with circulating decimals, the one pointed out in the propounded question being one of them. Of all the prime numbers 2 and 5 are the only ones that do not produce circulating decimals when divided into 1. When 1 is divided by a prime number N, 2 and 5 excepted, the resulting circulator contains either $N-1$ circulating decimals or a factor of $N-1$. In the former case the period is called full. Of all the prime numbers less than 100 7, 17, 19, 23, 29, 47, 59, 61, and 97 produce full periods, the periods containing respectively 6, 16, 18, 22, 28, 46, 58, 60, and 96 circulating decimals. All these have the property that the sum of any figure in the first half of the period and the corresponding figure in the second half is equal to 9. This remarkable property may be found thus:

Let N represent the prime number, a the first half of the period, and b the second, and let the period contain $2n$ figures. We have,

$$\frac{1}{N}=\frac{a}{10^m}+\frac{a}{10^{2m}}+\frac{b}{10^{4m}}+\ \ldots\ldots=\left(a.10^m+b\right)\left(\frac{1}{10^{2m}}+\frac{1}{10^{4m}}+\ \ldots\ldots\right)$$

$$=\frac{a.10^m+b}{10^{2m}-1}.\quad \text{But } \frac{1}{N}=\frac{a}{10^m}+\frac{N-1}{N.10^m},\text{ or}$$

$$\frac{b}{10^{2m}}+\frac{a}{10^{3m}}+\frac{b}{10^{4m}}+\ \ldots\ldots=\frac{N-1}{N.10^m}.$$

The first part of this equation is

$$=\frac{b.10^m+a}{10^{3m}}+\frac{b.10^m+a}{10^{5m}}+\ \ldots\ldots=\left(b.10^m+a\right)\left(\frac{1}{10^{3m}}+\frac{1}{10^{5m}}+\ \ldots\ldots\right)$$

$$=\frac{b.10^m+a}{10^m}\cdot\frac{1}{10^{2m}-1};\ \therefore\ \frac{b.10^m+a}{10^m}\cdot\frac{1}{10^{2m}-1}=\frac{N=1}{N.10^m},$$

$$\text{or } \frac{b.10^m+a}{10^{2m}-1}=1-\frac{1}{N}=1-\frac{a.10^m+b}{10^{2m}-1}.$$

By transposition $\frac{(a+b)10^m+(a+b)}{10^{2m}-1}=1$, or, $(a+b)(10^m+1)=10^{2m}-1$, or

$a+b=10^m-1$, but the number 10^m-1 must necessarily contain m 9's only.

If the period is not a full period, but the period contains an even number of figures, the above law still holds good.

Another property of circulation is that when the number that produces the circulator is multiplied by the period, the resulting product consists of 9's only. For, designating the period by A, we have $\frac{1}{N}=A+\frac{1}{N.10^m}$.

$\therefore \frac{10^m-1}{10^m}=AN$; but 10^m-1 contains only 9's.

A third property of circulation with full periods is, that, when $1/N$ is multiplied by any any integral number from 2 to $N-1$, the resulting periods will contain the same figures and in the same succession, only beginning with a different figure. This interesting property may be found as follows :

$$\text{Let } \frac{1}{N}=\frac{a}{10}+\frac{b}{10^2}+\frac{c}{10^3}+\frac{d}{10^4}+\dots\dots\frac{l}{10^{N-1}}+\frac{a}{10^N}+\frac{b}{10^{N+1}}+\dots\dots$$

Let us, for instance, denote the sum of the remaining fractions after the third by $\frac{r}{N.10^3}$, we have $\frac{1}{N}=\frac{a}{10}+\frac{b}{10^2}+\frac{c}{10^3}+\frac{r}{N.10^3}$, whence,

$$\frac{r}{N.10^3}=\frac{1}{N}-\frac{a}{10}-\frac{b}{10^2}-\frac{c}{10^3}=\frac{d}{10^4}+\frac{e}{10^5}+\dots\frac{l}{10^{N-1}}+\frac{a}{10^N}+\frac{b}{10^{N-1}}+\dots$$

Multiplying by 10^3, we have,

$$\frac{r}{N}=\frac{d}{10}+\frac{e}{10^2}+\dots\dots\frac{l}{10^{N-4}}+\frac{a}{10^{N-3}}+\frac{b}{10^{N-2}}+\dots\dots,$$

which proves the proposition.

If M and N are two different prime numbers, the number of figures in the period of the circulator resulting from converting $1/MN$ into a decimal fraction is the least common multiple of the number of figures in the periods of the circulator resulting from converting $1/M$ and $1/N$ into decimal fractions. There are some more properties of circulation, but space forbids me to mention them.

Note. **Prof. Nelson S. Roray calls attention to the error in the statement of No. 78; "minus 126" should be minus 125. The problem is found on page 344, Case II., example 4, Brooks' Higher Arithmetic. Dr. Brooks, in a letter to us, also called our attention to this error.**

Profs. P. S. Berg and Chas. C. Cross sent in solutions to problem 83 too late for credit in the November number.